1. APPLICATIONS OF MATRICES AND DETERMINANTS

TEST-1

1. If \( A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix} \), verify that \( A (\text{Adj} \ A) = (\text{Adj} \ A) A = | A | \).

2. Given \( A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & -1 \end{pmatrix} \) verify that \( | \text{Adj} \ A | = | A |^2 \).

3. Find the inverse of \( A = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix} \).

4. Find the inverse of \( A = \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \) and verify that \( AA^{-1} = I \).

5. If \( A = \begin{pmatrix} a_1 & 0 & a \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix} \) and none of the a’s are zero, find \( A^{-1} \).

6. If \( A = \begin{pmatrix} 4 & -3 & 4 \\ -1 & 2 & -2 \\ 0 & 0 & a_3 \end{pmatrix} \) show that the inverse of \( A \) is itself.

7. If \( A^{-1} = \begin{pmatrix} 1 & 3 & 4 \\ 3 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix} \) find \( A \).

8. Show that \( A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \) and \( B = \begin{pmatrix} \frac{1}{18} & \frac{-5}{18} & \frac{7}{18} \\ \frac{7}{18} & \frac{1}{18} & \frac{-5}{18} \\ \frac{-5}{18} & \frac{7}{18} & \frac{1}{18} \end{pmatrix} \) are inverse of each other.

TEST-2

9. Solve the equations \( x - 2y + 3z = 1 \), \( 3x - y + 4z = 3 \), \( 2x + y - 2z = -1 \) by matrix method.

10. Solve the equations \( 6x - 7y = 16 \), \( 9x - 5y = 35 \) by Cramer’s rule.

11. Two types of radio valves A, B are available and two types of radios P and Q are assembled in a small factory. The factory uses 2 valves of type A and 3 valves of type B for the type of radio P, and for the radio Q it uses 3 valves of type A and 4 valves of type B. If the number of valves of type A and B used by the factory are 130 and 180 respectively, find out the number of radios assembled. Use matrix method.

12. The cost of 2 kg. of wheat and 1 kg. of sugar is Rs. 7. The cost of 1 kg. wheat and 1 kg. of rice is Rs. 7. The cost of 3 kg. of wheat, 2 kg. of sugar and 1 kg. of rice is Rs. 7. Find the cost of each per kg., using matrix method.

13. There are three commodities X, Y and Z which are bought and sold by three dealers A, B and C. Dealer A purchases 2 units of X and 5 units of Z and sells 3 units of Y, dealer B purchases 5 units of X, 2 units of Y and sells 7 units of Z and dealer C purchases 3 units of Y, 1 unit of Z and sells 4 units of X. In the process A earns Rs. 11 and...
C Rs.5 but B loses Rs. 12. Find the price of each of the commodities X, Y and Z, by using determinants.

14. A company produces three products everyday. The total production on a certain day is 45 tons. It is found that the production of the third product exceeds the production of the first product by 8 tons while the total production of the first and third product is twice the production of second product. Determine the production level of each product by using Cramer’s rule

**TEST-3**

15. The data below are about an economy of two industries P and Q. The values are in millions of rupees.

<table>
<thead>
<tr>
<th>Producer</th>
<th>User</th>
<th>Final demand</th>
<th>Total output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P</td>
<td>Q</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>14</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Q</td>
<td>7</td>
<td>18</td>
<td>11</td>
</tr>
</tbody>
</table>

Determine the outputs if the final demand changes to 20 for P and 30 for Q.

16. Suppose the inter-relationship between the production of two industries P and Q in a year (in lakhs of rupees) is

<table>
<thead>
<tr>
<th>Producer</th>
<th>User</th>
<th>Final demand</th>
<th>Total output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P</td>
<td>Q</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>15</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Q</td>
<td>20</td>
<td>30</td>
<td>15</td>
</tr>
</tbody>
</table>

Find the outputs when the final demand changes to (i) 12 for P and 18 for Q (ii) 8 for P and 12 for Q.

17. In an economy of two industries P and Q the following table gives the supply and demand positions in millions of rupees.

<table>
<thead>
<tr>
<th>Producer</th>
<th>User</th>
<th>Final demand</th>
<th>Total output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P</td>
<td>Q</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>16</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>Q</td>
<td>8</td>
<td>40</td>
<td>32</td>
</tr>
</tbody>
</table>

Find the outputs when the final demand changes to 18 for P and 44 for Q.

18. The data below are about an economy of two industries P and Q. The values are in crores of rupees.

<table>
<thead>
<tr>
<th>Producer</th>
<th>User</th>
<th>Final demand</th>
<th>Total output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P</td>
<td>Q</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>50</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>Q</td>
<td>100</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

Find the outputs when the final demand changes to 300 for P and 600 for Q.

19. The inter - relationship between the production of two industries P and Q in crores of rupees is given below.

<table>
<thead>
<tr>
<th>Producer</th>
<th>User</th>
<th>Total output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P</td>
<td>Q</td>
</tr>
<tr>
<td>P</td>
<td>300</td>
<td>800</td>
</tr>
<tr>
<td>Q</td>
<td>600</td>
<td>200</td>
</tr>
</tbody>
</table>

If the level of final demand for the output of the two industries is 5,000 for P and 4,000 for Q, at what level of output should the two industries operate?

**TEST-4**

20. Two products A and B currently share the market with shares 60% and 40% each respectively. Each week some brand switching takes place. Of those who bought A the previous week, 70% buy it again whereas 30% switch over to B. Of those who bought B the previous week, 80% buy it again whereas 20% switch over to A. Find their shares after one week and after two weeks. If the price war continues, when is the equilibrium reached?

21. A new transit system has just gone into operation in a city. Of those who use the transit system this year, 10% will switch over to using their own car next year and 90% will continue to use the transit system. Of those who use their cars this year, 80% will continue to use their cars next year and 20% will switch over to the transit system. Suppose the population of the city remains constant and that 50% of the commuters use the transit system and 50% of the commuters use their own car this year,

(i) What percent of commuters will be using the transit system after one year?

(ii) What percent of commuters will be using the transit system in the long run?

22. Two products P and Q share the market currently with shares 70% and 30% each respectively. Each week some brand switching takes place. Of those who bought P the previous week, 80% buy it again whereas 20% switch over to Q. Of those who bought Q the previous week, 40% buy it again whereas 60% switch over to P. Find their shares after two weeks. If the price war continues, when is the equilibrium reached?

23. The subscription department of a magazine sends out a letter to a large mailing list inviting subscriptions for the magazine. Some of the people receiving this letter already subscribe to the magazine while others do not. From this mailing list, 60% of those who already subscribe will subscribe again while 25% of those who do not now subscribe will subscribe. On the last letter it was found that 40% of those receiving it ordered a subscription. What percent of those receiving the current letter can be expected to order a subscription?

24. Two newspapers A and B are published in a city. Their present market shares are 15% for A and 85% for B. Of those who bought A the previous year, 65% continue to buy it again while 35% switch over to B. Of those who bought B the previous year, 55% buy it again and 45% switch over to A. Find their market shares after two years.
2. ANALYTICAL GEOMETRY

TEST-1

1. Find the focus, latus rectum, vertex and directrix of the parabola
   \[ y^2 - 8x - 2y + 17 = 0. \]
2. Find the vertex, focus, axis, directrix and length of semilatus rectum of
   the parabola \[ 4y^2 + 12x - 20y + 67 = 0. \]
3. The girder of railway bridge is a parabola with its vertex at the highest point, which is 15 metres above the span of length 150 metres. Find its height 30 metres from the mid point.
4. Find the foci, latus recta, vertices and directrices of the following parabolas:
   (i) \[ y^2 + 4x - 2y + 3 = 0 \]
   (ii) \[ y^2 - 4x + 2y - 3 = 0 \]
   (iii) \[ y^2 - 8x - 9 = 0 \]
   (iv) \[ x^2 - 3y + 3 = 0 \]

TEST-2

5. Find the eccentricity, foci and latus rectum of the ellipse
   \[ 9x^2 + 16y^2 = 144. \]
6. Find the centre, eccentricity, foci and directrices of the ellipse
   \[ 3x^2 + 4y^2 - 6x + 8y - 5 = 0. \]
7. Find the centre, vertices, eccentricity, foci and latus rectum and
   directrices of the ellipse.
   (i) \[ 9x^2 + 4y^2 = 36 \]
   (ii) \[ 7x^2 + 4y^2 - 14x + 40y + 79 = 0 \]
   (iii) \[ 9x^2 + 16y^2 + 36x - 32y - 92 = 0 \]

TEST-3

8. Find the equations of the asymptotes of the hyperbola
   \[ 2x^2 + 5xy + 2y^2 - 11x - 7y - 4 = 0. \]
9. Find the centre, eccentricity, foci and latus rectum of the hyperbola
   \[ 9x^2 - 16y^2 - 18x - 64y - 199 = 0. \]
10. Find the equation to the hyperbola which has the line \[ x + 4y - 5 = 0 \]
    and \[ 2x - 3y + 1 = 0 \] for its asymptotes and which passes through the point (1, 2).
11. Find the centre, eccentricity, foci and directrices for the following hyperbolas
    (i) \[ 9x^2 - 16y^2 = 144 \]
    (ii) \[ \frac{(x+2)^2}{9} - \frac{(y+4)^2}{7} = 1 \]
    (iii) \[ 12x^2 - 4y^2 - 24x + 32y - 127 = 0 \]
12. Find the equation to the asymptotes of the hyperbola
    (i) \[ 3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0 \]
    (ii) \[ 8x^2 + 10xy - 3y^2 - 2x + 4y - 2 = 0 \]
13. Find the equation to the hyperbola which passes through (2,3) and
    has for its asymptotes the lines \[ 4x + 3y - 7 = 0 \]
    and \[ x - 2y = 1. \]
14. Find the equation to the hyperbola which has \[ 3x - 4y + 7 = 0 \]
    and \[ 4x + 3y + 1 = 0 \] for asymptotes and which passes through the origin.

3. APPLICATIONS OF DIFFERENTIATION – I

TEST-1

1. If \( y = \frac{1-2x}{2+3x} \) find \( \frac{dy}{dx} \). Obtain the values of \( \eta \) when \( x = 0 \) and \( x = 2 \).

2. If AR and MR denote the average and marginal revenue at any output level, show that elasticity of demand is equal to \( \frac{AR}{AR-\text{MR}} \). Verify this for the linear demand law \( p = a + bx \), where \( p \) is price and \( x \) is the quantity.

3. Find the elasticity of demand with respect to the price for the following demand functions.
   (i) \( p\sqrt{a} - bx \), a and b are constants (ii) \( x = \frac{a}{p^2} \)

4. A firm produces \( x \) tonnes of output at a total cost \( C(x) = Rs\left(\frac{1}{2}x^3 - 4x^2 + 25x + 8\right) \). Find (i) Average Cost (ii) Average Variable Cost and (iii) Average Fixed Cost. Also find the value of each of the above when the output level is 10 tonnes.

5. If the total cost \( C \) of making \( x \) tonnes of a product is \( C = 10 + 30\sqrt{x} \). Find the marginal cost at 100 tonnes output and find the level of output at which the marginal cost is Rs. 0.40 per ton.

6. The cost function for the production of \( x \) units of an item is given by \( C = Rs\left(\frac{1}{10}x^3 - 4x^2 + 8x + 4\right) \). Find (i) the average cost (ii) the marginal cost and (iii) the marginal average cost.

7. The price and quantity \( x \) of a commodity are related by the equation \( x = 30 - 4p - p^2 \). Find the elasticity of demand and marginal revenue.

8. The price and quantity \( q \) of a commodity are related by the equation \( q = 32 - 4p - p^2 \). Find the elasticity of demand and marginal revenue when \( p = 3 \).

TEST-2

9. The unit price, \( p \) of a product is related to the number of units sold, \( x \), by the demand equation \( p = 400 - \frac{x}{1000} \). The cost of producing \( x \) units is given by \( C(x) = 50x + 16,000 \). The number of units produced and sold, \( x \) is increasing at a rate of 200 units per week. When the number of units produced and sold is 10,000, determine the instantaneous rate of change with respect to time, \( t \) (in weeks) of (i) Revenue (ii) Cost (iii) Profit.

10. Given are the following revenue, cost and profit equations \( R = 800x - \frac{x^2}{10}, C = 40x + 5,000, P = R - C \), where \( x \) denotes the number of units produced and sold (per month). When the production is at 2000 units and increasing at the rate of 100 units per month, determine the instantaneous rate of change with respect to time, \( t \) (in months), of (i) Revenue (ii) Cost and (iii) Profit.

11. The unit price, \( p \) of some product is related to the number of units sold, \( x \), by the demand function \( p = 200 - \frac{x}{1000} \). The cost of producing \( x \) units of this product is given by \( C = 40x + 12,000 \). The number of units produced and sold \( x \) is increasing at the rate of 300 units per week. When the number of units produced and sold is 20,000 determine the instantaneous rate of change with respect to time, \( t \) (in weeks) of (i) Revenue (ii) Cost and (iii) Profit.

TEST-3

12. Find the equation of the tangent and normal to the supply curve \( y = x^2 + x + 2 \) when \( x = 6 \).

13. Find the equation of the tangent and normal to the demand curve \( y = 36 - x^2 \) when \( y = 11 \).

14. Find the equation of the tangent and normal to the curve \( y(x - 2) (x - 3) - x + 7 = 0 \) at the point where it cuts the x axis.

15. At what points on the circle \( x^2 + y^2 - 2x - 4y + 1 = 0 \), the tangent is parallel to (i) x-axis (ii) y-axis.

16. Find the equation of the tangent and normal at the point
(a sec \( \theta \), b tan \( \theta \)) on the hyperbola \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \).
17. Prove that the curves \( y = x^2 - 3x + 1 \) and \( x(y + 3) = 4 \) intersect at right angles at the point (2, -1).
18. Prove that \( \frac{x}{a} + \frac{y}{b} = 1 \) touches the curve \( y = be^{-x/a} \) at the point
where the curve cuts the y-axis.
19. Find the equations of the tangents and normals to the following
curves
(i) \( y = x^2 \log x \) at \( x = e \)
(ii) \( x = a \cos \theta, y = b \sin \theta \) at \( \theta = \frac{\pi}{4} \)
20. Find the points on the curve \( y = (x - 1)(x - 2) \) at which the
tangent makes an angle 135° with the positive direction of the x-axis.
21. Find the equations of the tangent and normal at the point
(a cos \( \theta \), b sin \( \theta \)) on the ellipse
\( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \).
22. Prove that for the cost function \( C = 100 + 2x + 10 \), where \( x \) is the
output, the slope of AC curve \( = \frac{1}{x} (MC - AC) \).
23. For the cost function \( y = 2x \left( \frac{x^4 + 4}{x^4} \right) + 3 \) prove that the marginal
cost falls continuously as the output \( x \) increases.

4. APPLICATIONS OF DIFFERENTIATION – II

Test-1

1. For the cost function \( C = 2000 + 1800x - 75x^2 + x^3 \) find
when the total cost (C) is increasing and when it is decreasing. Also
discuss the behaviour of the marginal cost (MC)
2. Find the absolute (global) maximum and minimum value of the
function \( f(x) = 3x^5 - 25x^3 + 60x + 1 \) in the interval \([ - \frac{3}{2}, 3] \).
3. What is the maximum slope of the tangent to the curve
\( y = -x^3 + 3x^2 + 9x - 27 \) and at what point is it?
4. Separate the intervals in which the function \( x^3 + 8x^2 + 5x - 2 \) is
increasing or decreasing.
5. For the following total revenue functions, find when the total
revenue (R) is increasing and when it is decreasing. Also discuss the
behaviour of marginal revenue (MR).
(i) \( R = 90 + 6x^2 - x^3 \)
(ii) \( R = -105x + 60x^2 - 5x^3 \)
6. For the following cost functions, find when the total cost (C) is
increasing and when it is decreasing. Also discuss the behaviour of
marginal cost (MC).
(i) \( C = 2000 + 600x - 45x^2 + x^3 \)
(ii) \( C = 200 + 40x - \frac{1}{2}x^2 \).
7. Find the maximum and minimum values of the function
(i) \( 2x^3 - 15x^2 + 24x - 15 \)
(ii) \( x^3 - 6x^2 + 9x + 15 \)
8. Find the absolute (global) maximum and minimum values of the
function \( f(x) = 3x^5 - 25x^3 + 60x + 15 \) in the interval
\([ - \frac{3}{2}, 3] \).
9. Show that the maximum value of the function
\( f(x) = x^2 - 27x + 108 \) is 108 more than the minimum value.
10. Find the intervals in which the curve
\( y = x^4 - 3x^3 + 3x^2 + 5x + 1 \) is convex upward and convex
downward.
11. Find the maximum and minimum values of the function
\( x^5 - 5x^4 + 5x^3 - 1 \). Discuss its nature at \( x = 0 \).
12. The total revenue (TR) for commodity \( x \) is \( TR = 12x + \frac{x^2}{2} - \frac{x^3}{3} \). Show
that at the highest point of average revenue (AR), AR = MR
(where MR = Marginal Revenue).

Test-2

13. The total cost and total revenue of a firm are given by
\[ C = x^3 - 12x^2 + 48x + 11 \text{ and } R = 83x - 4x^2 - 21. \] Find the output (i) when the revenue is maximum (ii) when profit is maximum.

14. If \( C = \frac{1}{3}x^3 - 5x^2 + 28x + 10 \) where \( x \) is the output. A tax at Rs.2 per unit of output is imposed and the producer adds it to his cost. If the market demand function is given by \( p = 2530 - 5x \), where Rs. \( p \) is the price per unit of output, find the profit maximising output and price.

15. A company uses annually 24,000 units of raw materials which costs Rs. 1.25 per unit, placing each order costs Rs. 22.50 and the holding cost is 5.4% per year of the average inventory. Find the EOQ, time between each order, total number of orders per year. Also verify that at EOQ carrying cost is equal to ordering cost.

16. A manufacturing company purchases 9000 parts of a machine for its annual requirements. Each part costs Rs.20. The ordering cost per order is Rs.15 and carrying charges are 15% of the average inventory. Find (i) economic order quantity (ii) time between each order (iii) minimum average cost.

17. A firm produces an output of \( x \) tons of a certain product at a total cost given by \( C = 300x - 10x^2 + \frac{1}{3}x^3 \). Find the output at which the average cost is least and the corresponding value of the average cost.

18. A firm produces \( x \) units of output per week at a total cost of Rs.(\( \frac{1}{3}x^3 - 2 + 5x + 3 \)). Find the level at which the marginal cost and the average variable cost attain their respective minimum.

19. It is known that in a mill the number of labourers \( x \) and the total cost \( C \) are related by \( C = \frac{3}{2(x-4)} + \frac{3}{32} \). What value of \( x \) will minimise the cost?

20. A manufacturer has to supply his customer with 600 units of his products per year. Shortages are not allowed and storage cost amounts to 60 paise per unit per year. When the set up cost is Rs. 80 find, (i) the economic order quantity. (ii) the minimum average yearly cost (iii) the optimum number of orders per year (iv) the optimum period of supply per optimum order.

21. The annual demand for an item is 3200 units. The unit cost is Rs.6 and inventory carrying charges 25% per annum. If the cost of one procurement is Rs.150, determine (i) Economic order quantity. (ii) Time between two consecutive orders (iii) Number of orders per year (iv) minimum average yearly cost.

22. Calculate the EOQ in units and total variable cost for the following items, assuming an ordering cost of Rs.5 and a holding cost of 10%.

<table>
<thead>
<tr>
<th>Item</th>
<th>Annual demand</th>
<th>Unit price (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>460 units</td>
<td>1.00</td>
</tr>
<tr>
<td>B</td>
<td>392 units</td>
<td>8.60</td>
</tr>
<tr>
<td>C</td>
<td>800 units</td>
<td>0.02</td>
</tr>
<tr>
<td>D</td>
<td>1500 units</td>
<td>0.52</td>
</tr>
</tbody>
</table>

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