BM UNIT 1 [ IMPORTANT 2 MARKS ]

1) If \( A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix} \) and \( B = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix} \), then find the rank of \( AB \) and the rank of \( BA \).

2) Solve the following system of equations by rank method
\( x + y + z = 9, 2x + 5y + 7z = 52, 2x - y - z = 0 \)

3) For what values of the parameter \( l \), will the following equations fail to have unique solution: \( 3x - y + lz = 1, 2x + y + z = 2, x + 2y - lz = -1 \) by rank method.

4) Solve the following equations by using Cramer's rule
\( 2x + 3y = 7, 3x + 5y = 9 \)

5) A commodity was produced by using 3 units of labour and 2 units of capital, the total cost is Rs 62. If the commodity had been produced by using 4 units of labour and one unit of capital, the cost is Rs 56. What is the cost per unit of labour and capital? (Use determinant method).

6) A total of Rs 8,600 was invested in two accounts. One account earned \( 4 \frac{3}{4} % \) annual interest and the other earned \( 6 \frac{1}{2} % \) annual interest. If the total interest for one year was Rs 431.25, how much was invested in each account? (Use determinant method).

7) The subscription department of a magazine sends out a letter to a large mailing list inviting subscriptions for the magazine. Some of the people receiving this letter already subscribe to the magazine while others do not. From this mailing list, 45% of those who already subscribe will subscribe again while 30% of those who do not now subscribe will subscribe. On the last letter, it was found that 40% of those receiving it ordered a subscription. What percent of those receiving the current letter can be expected to order a subscription?

8) Two types of soaps A and B are in the market. Their present market shares are 15% for A and 85% for B. Of those who bought A the previous year, 65% continue to buy it again while 35% switch over to B. Of those who bought B the previous year, 55% buy it again and 45% switch over to A. Find their market shares after one year and when is the equilibrium reached?

9) Two products A and B currently share the market with shares 50% and 50% each respectively. Each week some brand switching takes place. Of those who bought A the previous week, 60% buy it again whereas 40% switch over to B. Of those who bought B the previous week, 80% buy it again where as 20% switch over to A. Find...
their shares after one week and after two weeks. If the price war continues, when is the equilibrium reached?

10) Find the rank of the matrix \( A = \begin{pmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 0 \end{pmatrix} \)

11) Find the rank of the matrix \( A = \begin{pmatrix} 4 & 5 & 2 & 2 \\ 3 & 2 & 1 & 6 \\ 4 & 4 & 8 & 0 \end{pmatrix} \)

12) Examine the consistency of the system of equations: \( x+y+z=7, x+2y+3z=18, y+2z=6 \). 

13) Find \( k \) if the equations \( 2x+3y-z=5, 3x-y+4z=2, x+7y-6z=k \) are consistent.

14) Find \( k \) if the equations \( x+y+z=1, 3x-y-z=4, x+5y+5z=k \) are inconsistent.

15) Solve the equations \( x+2y+z=7, 2x-y+2z=4, x+y-2z=-1 \) by using Cramer’s rule

16) The cost of 2kg. of wheat and 1kg. of sugar is Rs 100. The cost of 1kg. of wheat and 1kg. of rice is Rs 80. The cost of 3kg. of wheat, 2kg. of sugar and 1kg of rice is Rs 220. Find the cost of each per kg., using Cramer’s rule.

17) Find the rank of the following matrices

\[
\begin{pmatrix} 1 & -1 \\ 3 & -6 \end{pmatrix}
\]

18) Find the rank of the following matrices

\[
\begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{pmatrix}
\]

19) Find the rank of the following matrices

\[
\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}
\]

20) Solve the following equation by using Cramer’s rule

\[
5x + 3y = 17; 3x + 7y = 31
\]

21) Find the rank of the matrix

\[
\begin{pmatrix} 7 & -1 \\ 2 & 1 \end{pmatrix}
\]

22) Find the rank of the matrix

\[
\begin{pmatrix} 2 & -4 \\ -1 & 2 \end{pmatrix}
\]

23) Solve \( x + 2y = 3 \) and \( x + y = 2 \) using Cramer’s rule.

24) Solve: \( x + 2y = 3 \) and \( 2x + 4y = 6 \) using rank method.

25) Show that the equations \( x + y + z = 6, x + 2y + 3z = 14 \) and \( x + 4y + 7z = 30 \) are consistent

26) Solve: \( 2x + 3y = 4 \) and \( 4x + 6y = 8 \) using Cramer’s rule.

27) If \( A \) and \( B \) are non-singular matrices, prove that \( AB \) is non-singular.

28) For what value of \( x \), the matrix

\[
A = \begin{pmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{pmatrix}
\]

is singular?
29) If \[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}
\]
find \(x, y\), and \(z\).

30) Two newspapers A and B are published in a city. Their market shares are 15% for A and 85% for B of those who bought A the previous year, 65% continue to buy it again while 35% switch over to B. Of those who bought B the previous year, 55% buy it again and 45% switch over to A. Find their market shares after one year.
1) Find the rank of the matrix \[
\begin{pmatrix}
5 & 3 & 0 \\
1 & 2 & -4 \\
-2 & -4 & 8 \\
1 & 2 & -1 \\
2 & 4 & 1 & -2 \\
3 & 6 & 3 & -7
\end{pmatrix}
\]

2) Find the rank of the matrix \[
\begin{pmatrix}
1 & 1 & 0 \\
2 & 2 & -1 \\
-4 & -4 & 8 \\
-2 & -1 & 3 \\
1 & 2 & -1 \\
2 & 4 & 1 & -2 \\
3 & 6 & 3 & -7
\end{pmatrix}
\]

3) Show that the equations \(x+y=5, 2x+y=8\) are consistent and solve them.

4) Show that the equations \(3x-2y=6, 6x-4y=10\) are inconsistent.

5) Show that the equations \(x+y+z=6, x+2y+3z=14, x+4y+7z=30\) are consistent and solve them.

6) Find \(k\), if the equations \(x+2y-3z=-2, 3x-2y+3z=18, y+kz=6\) are inconsistent.

7) Find \(k\), if the equations \(x+y+z=7, x+2y+3z=18, y+kz=6\) are inconsistent.

8) Investigate for what values of 'a' and 'b' the following system of equations \(x+y+z=6, x+2y+3z=10, x+2y+az = b\) have
   (i) no solution
   (ii) a unique solution
   (iii) an infinite number of solutions.

9) The total number of units produced (\(P\)) is a linear function of amount of over times in labour (in hours) \((l)\), amount of additional machine time (\(m\)) and fixed finishing time (\(a\))
   i.e., \(P = a + bl + cm\)

   From the data given below, find the values of constants \(a, b\) and \(c\)

<table>
<thead>
<tr>
<th>Day</th>
<th>Production (in Units P)</th>
<th>Labour (in Hrs l)</th>
<th>Additional Machine Time (in Hrs m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>6,950</td>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>Tuesday</td>
<td>6,725</td>
<td>35</td>
<td>9</td>
</tr>
<tr>
<td>Wednesday</td>
<td>7,100</td>
<td>40</td>
<td>12</td>
</tr>
</tbody>
</table>

Estimate the production when overtime in labour is 50 hrs and additional machine time is 15 hrs.

10) Consider the matrix of transition probabilities of a product available in the market in two brands A and B.
    \[
    \begin{pmatrix}
    A & B \\
    0.9 & 0.1 \\
    0.3 & 0.7
    \end{pmatrix}
    \]
    Determine the market share of each brand in equilibrium position.

11) Parithi is either sad (\(S\)) or happy (\(H\)) each day. If he is happy in one day, he is sad on the next day by four times out of five. If he is sad on one day, he is happy on the next day.
by two times out of three. Over a long run, what are the chances that Parithi is happy on any given day?

12) Akash bats according to the following traits. If he makes a hit (S), there is a 25% chance that he will make a hit his next time at bat. If he fails to hit (F), there is a 35% chance that he will make a hit his next time at bat. Find the transition probability matrix for the data and determine Akash’s long-range batting average.

13) 80% of students who do maths work during one study period, will do the maths work at the next study period. 30% of students who do english work during one study period, will do the english work at the next study period.

Initially there were 60 students do maths work and 40 students do english work.

Calculate,

(i) The transition probability matrix
(ii) The number of students who do maths work, english work for the next subsequent 2 study periods.

14) Find the rank of the matrix

\[ A = \begin{pmatrix} 2 & 4 & 5 \\ 4 & 8 & 10 \\ -6 & -12 & -15 \end{pmatrix} \]

15) Show that the equations \(2x - Y + z = 7, 3x + y - 5z = 13, x + y + z = 5\) are consistent and have a unique solution.

16) Show that the equations \(x + 2y = 3, Y - z = 2, x + y + z = 1\) are consistent and have infinite sets of solution.

17) Show that the equations \(x - 3y + 4z = 3, 2x - 5y + 7z = 6, 3x - 8y + 11z = 1\) are inconsistent.

18) If \( A = \begin{pmatrix} 2 & 4 \\ 4 & 3 \end{pmatrix} \), \( X = \begin{pmatrix} n \\ 1 \end{pmatrix} \) \( B = \begin{pmatrix} 8 \\ 11 \end{pmatrix} \) and \( AX = B \) then find \( n \).

19) Solve: \(2x + 3y = 5, 6x + 5y = 11\)

20) Two products A and B currently share the market with shares 60% and 40% each respectively. Each week some brand switching latees place. Of those who bought A the previous week 70% buy it again whereas 30% switch over to B. Of those who bought B the previous week, 80% buy it again whereas 20% switch over to A. Find their shares after one week and after two weeks.

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BM UNIT 1 [IMPORTANT 5 MARKS]

Date: 21-Jun-19

1) Solve the equations \(2x + 3y = 7, 3x + 5y = 9\) by Cramer’s rule.

2) The following table represents the number of shares of two companies A and B during the month of January and February and it also gives the amount in rupees invested by Ravi during these two months for the purchase of shares of two companies. Find the price per share of A and B purchased during both the months.

<table>
<thead>
<tr>
<th>Months</th>
<th>Number of Shares of the company</th>
<th>Amount invested by Ravi (in Rs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>A 10</td>
<td>B 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>125</td>
</tr>
<tr>
<td>February</td>
<td>A 9</td>
<td>B 12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150</td>
</tr>
</tbody>
</table>

3) The total cost of 11 pencils and 3 erasers is Rs 64 and the total cost of 8 pencils and 3 erasers is Rs 49. Find the cost of each pencil and each eraser by Cramer’s rule.

4) Solve by Cramer’s rule \(x+y+z=4,2x−y+3z=1,3x+2y−z = 1\)


6) An automobile company uses three types of Steel \(S_1, S_2, S_3\) and \(S_4\) for providing three different types of Cars \(C_1, C_2, C_3\). Steel requirement \(R\) (in tonnes) for each type of car and total available steel of all the three types are summarized in the following table.

<table>
<thead>
<tr>
<th>Types of Steel</th>
<th>Types of Car</th>
<th>Total Steel available</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_1)</td>
<td>(C_1) 3</td>
<td>(C_2) 2</td>
</tr>
<tr>
<td>(S_2)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(S_3)</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Determine the number of Cars of each type which can be produced by Cramer’s rule.

7) A mixture is to be made of three foods A, B, C. The three foods A, B, C contain nutrients \(p, Q, R\) as shown below:

<table>
<thead>
<tr>
<th>Ounces per pound of Nutrient</th>
<th>Food</th>
<th>(P)</th>
<th>(Q)</th>
<th>(R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
\[ \begin{vmatrix} c & 4 & 1 \\ 2 & 3 & 0 \\ 1 & 1 & 1 \end{vmatrix} \]

How to form a mixture which will have 8 ounces of P, 5 ounces of Q and 7 ounces of R? (Cramer’s rule).

8) For what values of \( k \), the system of equations
   \[ \begin{align*}
   kx + y + z &= 1, \\
   ky + z &= 1, \\
   x + y + k &= 1 
   \end{align*} \]
   have
   (i) Unique solution
   (ii) More than one solution
   (iii) no solution

9) Using determinants, find the quadratic defined by \( f(x) = ax^2 + bx + c \) if \( f(1) = 0, f(2) = -2 \) and \( f(3) = -6 \).

10) A new transit system has just gone into operation in a city. Of those who use the transit system this year, 10% will switch over to using their own car next year and 90% will continue to use the transit system. Of those who use their cars this year, 80% will continue to use their cars next year and 20% will switch over to the transit system. Suppose the population of the city remains constant and that 50% of the commuters use the transit system and 50% of the commuters use their own car this year,
   (i) What percent of commuters will be using the transit system after one year?
   (ii) What percent of commuters will be using the transit system in the long run?

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