TWO MARKS \[ 5 \times 2 = 10 \]
1. \(2x+5y=-2,\ x+2y=-3\) (inverse method)
2. 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.
3. \(2x-y=7,\ 3x-2y=11\) (inverse method)
4. A total of 820 tickets were sold for a game for a total of Rs.9128. If adult tickets were sold for Rs. 12.00 and children ticket were sold for Rs.8.00. How many of each kind of tickets were sold (cramar’s method)
5. Mr. Guru had Rs.20,000 to invest. He invested part at 3% and rest at 2%. After one year he earned Rs.520 (cramar’s method)

THREE MARKS \[ 5 \times 3 = 15 \]
1. 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.
2. A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadas is Rs.150. The cost of the two dosai, two idlies and four vadas is Rs. 200. The cost of five dosai, four idlies and two vadas is Rs. 250. The family has Rs. 350 in hand and they ate 3 dosai and six idlies and six vadas. Will they be able to manage to pay the bill
1. \( \frac{1}{x} + \frac{2}{y} - \frac{1}{z} = 1 \), \( \frac{2}{x} + \frac{4}{y} + \frac{1}{z} = 5 \), \( \frac{3}{x} - \frac{2}{y} - \frac{2}{z} = 0 \) (cramer's method)

2. The prices of three commodities \( AB \), and \( C \) are Rs.\( x \), \( y \), and \( z \) per units respectively. A person \( P \) purchases 4 units of \( B \) and sells two units of \( A \) and 5 units of \( C \). Person \( Q \) purchases 2 units of \( C \) and sells 3 units of \( A \) and one unit of \( B \). Person \( R \) purchases one unit of \( A \) and sells 3 unit of \( B \) and one unit of \( C \). In the process, \( P,Q \) and \( R \) earn Rs.15,000, Rs.1,000 and Rs.4,000 respectively. Find the prices per unit of \( AB \), and \( C \). (inverse method)

3. In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is \( Y = ax^2 + bx + c \) with respect to a \( xy \)-coordinate system in the vertical plane and the ball traversed through the points \((10,8),(20,16),(30,18)\), can you conclude that Chennai Super Kings won the match? Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is \((70,0)\) (cramer's method)
i. Answer the following questions 5x2=10
1. 2x-2y+3z=2 , x+2y-z=3 , 3x+y-2z=1
2. 2x+2y+z=5 , x-y+z=1 , 3x+y+2z=4
3. x+2y+3z=0 , 3x+4y+4z=0 , 7x+10y+12z=0
4. 3x+2y+7z=0 , 4x-3y-2z=0 , 5x+9y+23z=0
5. x-y+z=-9 , 2x-2y+2z=-18 , 3x-3y+3z+27=0

ii. Answer the following questions 5x3=15
1. Determine the values of λ for which the following system of equations (3λ-8)x+3y+3z = 0 , 3x+(3λ-8)+3z=0 , 3x+3y+(3λ-8) = 0
2. x+3y-2z=0 , 2x-y+4z=0 , x-11y+14z=0
3. Find the condition on ab , and c so that the following system of linear equations has one parameter family of solutions: x+3y+2z=a , x+2y+3z = b , x+3x+y+7z=c
4. If ax²+bx+c is divided by x=3,x-5 and x-1 the remainders are 21.61 and 9 respectively. Find a,b & c (Use Gaussian elimination method.)
5. A boy is walking along the path y = ax²+bx+c through the points (-6,8) ,(-2,-12) , and (3,8). He wants to meet his friend at P(7,60). Will he meet his friend? (Use Gaussian elimination method.)

iii. Answer the following questions 5x5=25
1. An amount of Rs.65,000 is invested in three bonds at the rates of 6%, 8% and 9% per annum respectively. The total annual income is Rs.4,800. The income from the third bond is Rs. 600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.)
2. Find λ & μ values for x+2y+z=7 , x+y+ λz=μ, x +3y-5z=5 i) no soln ii) unique soln iii) infinite soln
3. Find the value of k for which the equations
   kx-2y+z=1, x-2ky+z=2 , x-2y+kz=1
   have (i) no solution (ii) unique solution (iii) infinitely many solutions
4. By using Gaussian elimination method, balance the chemical reaction equation:
   C₃H₈ + O₂ → CO₂ + H₂O
5. If the system of equations ax+y+z = 0 ,
   x+by+z=0 , x+y+cz = 0 , where ( a , b, c ≠ 1 ) has non trivial soln , then find the value of
   \[ \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} \]
6. If the system of equations ax+y+z = 0 ,
   x+by+z=0 , x+y+cz = 0 , where ( a , b, c ≠ 1 ) has non trivial soln , then find the value of
   \[ \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} \]


Padasalai

SAIVEERA ACADEMY
PEELAMEDU – 8098850809
MATHS 1ST CHAPTER FULL TEST

Marks : 80
Time : 1hrs 45 mins

1. CHOOSE THE BEST ANSWERS

1. If A is a 3×3 non-singular matrix such that $A^T A = A^T A & B = A^{-1} A^T$, then $BB^T =$

(1) $A$ (2) $B$ (3) $I$ (4) $B^T$

2. If $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ and $B = \text{adj } A & C = 3A$, then $| \text{adj } B | / | C | =$

(1) $1/3$ (2) $1/9$ (3) $1/4$ (4) $1$

3. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -1 & -2 & -3 \end{bmatrix}$ is

(1) $1$ (2) $2$ (3) $3$ (4) $4$

4. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ such that $\lambda A^{-1} = A$, then $\lambda$ is

(1) $7$ (2) $14$ (3) $19$ (4) $21$

5. If $\rho (A) \neq \rho (A | B)$, then system of equations

(1) consistent and has a unique solution
(2) consistent
(3) consistent and has infinitely many solutions
(4) inconsistent

6. If $A = \begin{bmatrix} 3 & x \\ -4 & 3 \end{bmatrix}$, then the value of $x$ is

(1) $4$ (2) $-3$ (3) $-4$ (4) $3$

7. If $A^{-1} A^T$ is symmetric then $A =$

(1) $A$ (2) $(A^T)^2$ (3) $I$ (4) $A^T$

8. If $A = \begin{bmatrix} \cos \theta & x \\ \sin \theta & \cos \theta \end{bmatrix}$ is orthogonal then value of $x$ is

(1) $\cos \theta$ (2) $-\cos \theta$ (3) $\sin \theta$ (4) $-\sin \theta$

9. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & -3 \end{bmatrix}$ is

(1) $1$ (2) $2$ (3) $3$ (4) $4$

10. If $P = \begin{bmatrix} 1 & x \\ 1 & 3 \\ 2 & 4 \end{bmatrix}$ is the adjoint of $3 \times 3$ matrix, $A$ and $|A| = 4$, then $x$ is

(1) $15$ (2) $12$ (3) $14$ (4) $11$

II. ANSWER THE FOLLOWING QUESTIONS

Q.no 18 is compulsory

11. If $\text{adj } A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ find $A^{-1}$

12. $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ given that $A^2 - 9A + 14I_2$ find $A^{-1}$ (NOT BY DIRECT METHOD)

13. Find inverse by Gauss Jordan method $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$

14. Solve by cramer’s method $5x - 2y + 16 = 0, x + 3y - 7 = 0$

15. Find out value of $\lambda$ and $\mu$ if matrix $0 \begin{bmatrix} 1 & -6 & -2 \\ 0 & 0 & \lambda - \mu + 8 \end{bmatrix}$

16. Solve by gaussian elimination $2x - y + z = 2, 6x - 3y + 3z = 6, 4x - 2y + 2z = 4$

17. Prove that if $\lambda = 2/3$ given matrix is non trivial solution

$(3\lambda - 8)x + 3y + 3z = 0, 3x + (3\lambda - 8) + 3z = 0, 3x + 3y + (3\lambda - 8) = 0$
18. If A and B are non-singular matrices of the same order, then the product AB is also non-singular and then prove that \((AB)^{-1} = B^{-1}A^{-1}\)

### III. Answer the following questions

**Q.no 27 is compulsory**

**6x3=18**

19. If \(A = \frac{1}{7}\) \(\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}\) is orthogonal, then find a, b, c

20. \(A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\) then prove that \(A(\text{adj}A) = (\text{adj} A)A = | A |\)

21. Find the rank of the matrix \(\begin{bmatrix} -3 & 4 & -2 & -1 \\ 6 & 2 & -7 & 7 \end{bmatrix}\) by reducing it to an echelon form

22. 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of two woman alone to finish the same work by using matrix inversion method.

23. A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself? (Use Cramer’s rule to solve the problem)

24. If \(ax^2 + bx + c\) is divided by \(x + 3, x - 5\) and \(x - 1\) the remainders are 21, 61 and 9 respectively. Find a, b & c (Use Gaussian elimination method.)

25. Solve by gaussian elimination method \(x - y + z = -9\), \(2x - 2y + 2z = -18\), \(3x - y + 3z + 27 = 0\)

26. By using Gaussian elimination method, balance the chemical reaction equation:

\[\text{C}_3\text{H}_6 + \text{O}_2 \rightarrow \text{CO}_2 + \text{H}_2\text{O}\]

27. Given \(A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}\), \(B = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}\) & \(C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\) Find a matrix \(X\) such that \(AXB = C\)

### IV. Answer all the questions

**8x5=40**

28. \(A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}\) show that \(A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}\)

29. If \(F(\alpha) = \begin{bmatrix} \cos \alpha & 0 \\ 0 & \sin \alpha \end{bmatrix}\) show that \(F(-\alpha) = [F(\alpha)]^{-1}\)

30. If \(A = \begin{bmatrix} -4 & 4 & 4 \\ -\sin \alpha & 0 & \cos \alpha \\ 5 & -3 & -1 \end{bmatrix}\) & \(B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\) find product \(AB\) & \(BA\) and solve the equation \(x - y + z = 4\), \(x - 2y - 2z = 9\), \(2x + y + 3z = 1\)

31. In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball travelled along a path in a vertical plane and the equation of the path is \(y = ax^2 + bx + c\) with respect to a xy-coordinate system in the vertical plane and the ball traversed through the points \((10,8),(20,16),(40,22)\)

Can you conclude that Chennai Super Kings won the match? Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is \((70,0)\))

32. The upward speed \(v(t)\) of a rocket at time \(t\) is approximated by \(v(t) = at^2 + bt + c\)

It has been found that the speed at times \(t = 3\), \(t = 6\) and \(t = 9\) seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time \(t = 15\) seconds. (Use Gaussian elimination method.)

33. Test for consistency of the following system of linear equations and if possible solve:

\[X + 2y + z = 3\], \(3x - y + 2z = 1\), \(x - 2y + 3z = 3\), \(x - y + z = 1\)

34. Investigate the values of \(\lambda\) and \(\mu\) the system of linear equations \(2x + 3y + 5z = 9\), \(7x + 3y - 5z = 8\), \(2x + 3y + \lambda z = \mu (i)\) no solution (ii) a unique solution (iii) an infinite number of solutions

35. If the system of equations \(px + qy + rz = 0\), \(ax + qy + cz = 0\), \(ax + qy + rz = 0\), where \((p \neq a, q \neq b, r \neq c)\) has non trivial soln, then find the value of \(p/p - a + q/q - b + r/r - c\)

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**SAIVEERA ACADEMY**

**REVOLUTION FOR LEARNING - 8098850809**
II. Answer the following  7 × 2 = 14

6. Simplify i, i², ...... i^{38}

7. Find the real and imaginary parts 3i^{20} - i^{19} / 2i - 1

8. Given the complex number z = 2 + 3i, represent the complex numbers in Argand diagram.
   (i) z, iz , , and z+ iz

9. Evaluate the following if z = 5- 2i and w = -1+ 3i
   i. z² + 2w
   ii. (z-w)²

10. If z₁ = 1-2i , z₂ = 2+5i find its additive and multiplicative inverse and z₁/z₂ , z₁.z₂

11. Find the least positive integer (1+i / 1-i)ⁿ = 1

12. Express it in rectangular form \( \frac{5+5i}{3-4i} \) & find its real and imaginary parts

III. Answer the following  7 × 3 = 21

11. Find the value of the real numbers x and y, if the complex number (2 + i)x + (1- i)y + 2 -3i and x +(-1 + 2i)y+1+i are equal
12. Simplify $\sum_{i=1}^{102}(i^n)$
13. Prove that $z$ is real if and only if $z = \overline{z}$
$z$ is purely imaginary if and only if $z = -\overline{z}$
14. If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$ find modulus of $z$
15. Show that $(2+i\sqrt{3})^2 - (2-i\sqrt{3})^2$ is purely imaginary
16. Find the least value of the positive integer $n$ for which $(\sqrt{3} + i)^n$
   i. real  ii. imaginary
17. Show that the equation $z^2 = \overline{z}$ has four solutions
18. Find the square roots of (i) $4 + 3i$
19. If the area of the triangle formed by the vertices $z_1z_2$, $z_3$ and $z + iz$ is 50 square units, find the value of modulus of $z$
20. If $|z| = 3$ show that $7 \leq |z + 6 - 8i| \leq 13$
21. For what values of $x$ and $y$, the numbers $-3 + ix^2$ and $x^2 + y + 4i$ are complex conjugate of each other

IV. Answer the following  

21. If $z_1, z_2, z_3$ three complex numbers such that modulus of $z_1 + z_2 + z_3$ is 5  $|z_1| = 1$ , $|z_2| = 2$, $|z_3| = 3$
   Show that modulus of $9z_1z_2 + 4z_1z_3 + z_1z_3$ is 6
22. If $z_1, z_2, z_3$ be complex numbers such that $|z_1| = r$, $|z_2| = r$, $|z_3| = r$
   Prove that Modulus of $z_1z_2 + z_1z_3 + z_1z_3 / z_1 + z_2 + z_3 = r$
23. Show that the equation $z^3 + 2\overline{z}$ = 0 has five solutions
24. Show that the points 7+9i, -3+7i, 3+3i form a right angled triangle on the argand diagram
25. Show that the points 2i, 1+i, 4+4i, 3+5i on the argand diagram forms a vertices of triangle
I. Answer the following 10 × 2 = 20
1. If \( \frac{z-4i}{z+4i} = 1 \) prove that locus of \( z \) is real axis
2. Find cartesian form of \( z = z^{-1} \)
3. Show that following equations form circle and find its radius and centre
   \( |z - 2 - i| = 3 \)
4. Obtain cartesian form \( |z - 4| = 3 \)
5. Find modulus & argument of \(-\sqrt{3} - i\)
6. Find principal argument
   \( z = -2 / 1+i\sqrt{3} \)
7. If \( z_1 = r_1(\cos \Theta_1 + i\sin \Theta_1) \) and
   \( z_2 = r_2(\cos \Theta_2 + i\sin \Theta_2) \)
   Then prove that
   \( z_1/z_2 = r_1/r_2(\cos (\Theta_1 - \Theta_2) + i\sin (\Theta_1 - \Theta_2)) \)
8. Simplify \( \frac{(1+\cos 2\Theta + i\sin 2\Theta)^{15}}{(1+\cos 2\Theta - i\sin 2\Theta)} \)
9. \( x = a+b \), \( y = a\omega + b\omega^2 \),
   \( z = a\omega^2 + b\omega \) show that \( xyz = a^3+b^3 \)
10. Solve \( x^4 + 4 = 0 \)

II. Answer the following 10 × 3 = 30
11. Find the roots of \( 4\sqrt{-1} \)
12. Solve \( z^3 + 27 = 0 \)
13. \( 2\cos \alpha = x + 1/x \) and \( 2\cos \beta = y + 1/y \) then show that
   \( x^m y^n + 1/ x^m y^n = 2 \cos (m\alpha + n\beta) \)
14. Find cube root of unity
15. Find all cube roots of \( \sqrt{3} + i \)
16. Simplify \( (-\sqrt{3} + 3i)^3 \)
17. Find rectangular form 
\[ \frac{\cos \frac{\pi}{6} - i \sin \frac{\pi}{6}}{2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})} \]
18. Find locus if 
\[ \arg \left( \frac{z - 1}{z + 3} \right) = \frac{\pi}{2} \]
19. Express in polar form 
\[ -1 + i \sqrt{3} \]
20. i. If \((1+i) (1+2i) (1+3i) \cdots (1+ni) = x+iy\) 
Show that \(2.5.10 \cdots (1+n^2) = x^2 + y^2 \) 
ii. If \(z^2 = (0,1)\) find \(z\)

III. Answer the following \(7 \times 5 = 35\)
21. Find locus \(\text{Im} \left[ \frac{2z+1}{iz+1} \right] = -2\)
22. If \(\alpha\) and \(\beta\) are roots of \(x^2-2x+2 = 0\) 
and \(\cot \Theta = y+1\) 
Then show that \((y + \alpha)^n - (y + \beta)^n / \alpha - \beta = \sin n\Theta / \sin n \Theta\)
23. If \(\alpha\) and \(\beta\) are roots of \(x^2-2x+4 = 0\) 
prove that \(\alpha^n - \beta^n = i 2^{n+1} \sin n\pi/3\)
24. Find sixth roots of unity 
25. Solve \(z^3 + 8i = 0\)
26. Show that \(\left( \frac{\sqrt{3}}{2} + \frac{i}{2} \right)^5 + \left( \frac{\sqrt{3}}{2} - \frac{i}{2} \right)^5 = -\sqrt{3}\)
27. Find quotient in rectangular form 
\[ \frac{2(\cos \frac{9\pi}{4} - i \sin \frac{9\pi}{4})}{4(\cos \frac{-3\pi}{2} + i \sin \frac{-3\pi}{2})} \]

***SAIVEERA ACADEMY- 8098850809*****
I. Choose the best answers

1. A zero of \( x^2 + 64 \) is
   (1) -8      (2) 4      (3) 4i      (4) -4

2. According to the rational root theorem, which number is not possible rational root of \( 2x^3 + 3x^2 + 2x + 3 \)
   (1) -3/2    (2) 3/1    (3) 1/3    (4) 2/3

3. The number of real numbers in \([0, 2\pi]\) satisfying \( 2\cos^2x - 9\cos x + 4 = 0 \)
   (1) 0       (2) 4      (3) 1/2    (4) -4

4. The polynomial \( x^5 - 19x^4 + 2x^3 + 5x^2 + 11 \) has
   (1) one negative and two real roots
   (2) one positive and two imaginary roots
   (3) atmost two positive roots and one negative root
   (4) no solution

5. The quadric equation whose roots are \( \alpha \) & \( \beta \) is
   (1) \( (x - \alpha)(x - \beta) = 0 \)
   (2) \( (x - \alpha)(x + \beta) = 0 \)
   (3) \( \alpha + \beta = \frac{b}{a} \)
   (4) \( \alpha \beta = -\frac{c}{a} \)

6. Let \( a > 0 \), \( b > 0 \), \( c > 0 \). Then both the roots of equation \( ax^2 + bx + c = 0 \) are
   (1) real and negative
   (2) real and positive
   (3) rational numbers
   (4) none

7. If the product of two roots of \( 3x^2 - 16x + 23 = 0 \) is 1, then the roots are
   (1) 4, -4, 6
   (2) 8, -8, 4
   (3) 6, -6, 4
   (4) -3, 1/3, 2

8. If \( \alpha, \gamma, \beta, \delta \) are roots of \( 2x^4 + 5x^3 - 7x^2 + 8 = 0 \) then \( \alpha \gamma \beta \delta \) is
   (1) -4
   (2) 4
   (3) 1
   (4) -1

9. One root of equation \( (12-x)(6x-1)(4x-1)(3x-1) = 5 \) is
   (1) \( \frac{1}{2} \)
   (2) \( -1/12 \)
   (3) \( 7/24 \)
   (4) \( 24/7 \)

10. The value of \( x \) which satisfy the equation \( x^2 + 3x + 2 = 0 \) is
    (1) -8
    (2) 4
    (3) 4i
    (4) -1

II. Answer all the questions

11. Construct a cubic equation with roots 2, -2, and 4

12. If \( \alpha \), \( \beta \) and \( \gamma \) are the roots of the cubic equation \( x^3 + 2x^2 + 3x + 4 = 0 \) form a cubic equation whose roots are \( 1/\alpha \), \( 1/\beta \), \( 1/\gamma \)

13. If \( \alpha \) and \( \beta \) are the roots of the quadratic equation \( 2x^2 - 7x + 13 = 0 \) construct a quadratic equation whose roots are \( \alpha^2 \) and \( \beta^2 \)

14. If \( \alpha \), \( \beta \), and \( \gamma \) are the roots of the equation \( x^3 + px^2 + qx + r = 0 \), find the value of \( \sum \frac{1}{\alpha \beta} \) in terms of coefficients

15. If \( p \) is real, discuss the nature of the roots of the equation \( 4x^2 + 4px + p^2 + 2 = 0 \) in terms of \( p \)

16. A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away.
   Formulate this into a mathematical problem to find the height of the part which was cut away
17. Form a polynomial equation with integer coefficients with \( \sqrt{\frac{2}{3}} \) as a root

18. Show that, if \( p, q, r \) are rational, the roots of the equation \( x^2 - 2px + p^2 - q^2 - r^2 + 2qr = 0 \)

19. Prove that a line cannot intersect a circle at more than two points

20. Find a polynomial equation of minimum degree with rational coefficients, having \( 2i + 3 \) as a root.

21. Solve the equation \( x^4 - 9x^2 + 20 = 0 \)

22. Solve \( 2x^3 + 11x^2 - 9x - 18 = 0 \)

23. Solve equation \( 2x^3 - x^2 - 18x + 9 = 0 \) if sum of two of its roots vanishes

24. Solve the equation \( 7x^3 - 43x^2 = 43x - 7 \)

25. Show that the polynomial \( 9x^9 + 2x^5 - x^4 - 7x^2 + 2 \) has at least six imaginary roots

III. Answer all the questions

26. Solve \( 8x^{2n} - 8x^{-2n} = 63 \)

27. \( \sqrt[3]{\frac{3}{a} + 3} \) \( \frac{a}{\sqrt[3]{x}} = \frac{b}{a} + \frac{6a}{b} \)

28. Find all real numbers satisfying \( 4^x - 3(2^{x^2}) + 2^5 = 0 \)

29. Solve equation \( x^4 - 10x^3 + 26x^2 - 10x + 1 = 0 \)

30. Solve; \((2x^2-1)(x^3+3)(2x+3)(x^2-2)+20=0\)

31. Solve the equation \( 9x^3 - 36x^2 + 44x - 16 = 0 \) if the roots form an arithmetic progression

32. If the roots of \( x^3 + px^2 + qx + r = 0 \) are in H.P. Prove that \( 9pqr = 27r^3 + 2p \)

33. Find the condition that the roots of \( ax^3 + bx^2 + cx + d = 0 \) are in geometric progression. Assume \( a, b, c, d \neq 0 \)

34. If \( 2 + i \) and \( 3 - \sqrt{2} \) are roots of the equation \( x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0 \), find all the roots

35. If a complex number \( z_0 \) is a root of a polynomial equation with real coefficients, then its complex conjugate \( \bar{z}_0 \) is also a root.

IV. Answer all the questions

36. Find the sum of the squares of the roots of \( ax^4 + bx^3 + cx^2 + dx + e = 0 \)

37. Form the equation whose roots are the squares of the roots of the cubic equation \( x^3 + ax^2 + bx + c = 0 \)

38. If \( p \) and \( q \) are roots of equation \( kx^2 + nx + n = 0 \) then show that \( \frac{p}{\sqrt{q}} + \frac{q}{\sqrt{p}} + \sqrt{n} = 0 \)

39. Solve the equation \( x^3 - 9x^2 + 14x + 24 = 0 \)

   if it is given that two of its roots are in the ratio \( 3 : 2 \)

40. \( \alpha, \beta, \gamma \) and \( \delta \) are the roots of the polynomial equation \( 2x^4 + 5x^3 - 7x^2 + 8 = 0 \), find a quadratic equation with integer coefficients whose roots are \( \alpha + \beta + \gamma + \delta \) and \( \alpha \beta \gamma \delta \)

41. If the equations \( x^2 + px + q = 0 \), \( x^2 + p'x + q' = 0 \) have a common root, show that it must be equal to \( \frac{pq' - p'q}{q-q'} \) or \( \frac{q-q'}{p'-p} \)

42. Determine \( k \) and solve the equation \( 2x^3 - 6x^2 + 3x + k = 0 \) if one of its roots is twice the sum of the other two roots.

***************SAIVEERA ACADEMY 8098850809***************
SECTION – A

1. If $p + qi = (2-3i)(4+2i)$ then value of $p$ is
   a) $14$  
   b) $-14$  
   c) $8$  
   d) $-8$

2. The points $i, -2+i, 1$ is near from the origin?
   a) $i$  
   b) $3$  
   c) $-2+i$  
   d) none of these

3. The value of $(\sin \frac{\pi}{6} + i \cos \frac{\pi}{6})^4$ is
   a) $0$  
   b) $1$  
   c) $i$  
   d) $1$

4. The conjugate of $i^2 + i^3 + i^4 + i^5 + i^6$ is
   a) $1$  
   b) $-1$  
   c) $0$  
   d) $-i$

5. Identify the wrong statement
   a) All the $n$ roots of $n$ th roots unity are in Geometrical Progression
   b) Sum of the $n$ roots of $n$ th roots unity is always equal to zero.
   c) Product of the $n$ roots of $n$ th roots unity is equal to $(−1)^{n−1}$
   d) $n$ roots have arguments that differ by $\frac{2\pi}{n}$

6. If $z_n = \cos n\pi/3 + i \sin n\pi/3$, then $z_1$ $z_2$ $z_3$ ...
   a) $1$  
   b) $1$  
   c) $i$  
   d) $-i$

7. $|z| = 2$ represents a circle, the radius of circle is
   a) $-1$  
   b) $2$  
   c) $4$  
   d) $1$

8. If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$, modulus of $9z_1z_2 + 4z_1z_3 + z_1z_3$ is $36$ then $|z_1 + z_2 + z_3|$ is
   a) $2$  
   b) $6$  
   c) $4$  
   d) $1$

9. The value of $e^{i\theta} + e^{-i\theta}$ is
   a) $\sin \theta$  
   b) $-\sin \theta$  
   c) $2i\sin \theta$  
   d) $2\cos \theta$

10. The polar form of the complex number $(i^{25})^3$ is
    a) $\cos \pi/2 + i \sin \pi/2$  
    b) $\cos \pi + i \sin \pi$  
    c) $\cos \pi - i \sin \pi$  
    d) $\cos \pi/2 - i \sin \pi/2$

11. The value of $\left( \frac{-1 + \sqrt{3}i}{2} \right)^{200} + \left( \frac{-1 - \sqrt{3}i}{2} \right)^{200}$ is
    a) $2$  
    b) $0$  
    c) $-1$  
    d) $1$

12. If the amplitude of a complex number is $\pi$ then the number is
    (1) purely imaginary  
    (2) purely real  
    (3) $0$  
    (4) neither real nor imaginary

13. The principal value of $\arg Z$ lies in the interval
    a) $[0, \pi/2]$  
    b) $(-\pi, \pi]$  
    c) $[0, -\pi/2]$  
    d) $(-\pi/2, \pi/2]$
14. If $\arg(z-i/z+2) = \pi/4$ then locus of $z$ is
   a) ellipse     b) parabola     c) hyperbola     d) circle

15. Identify the wrong identity
   a) $\arg(z_1 z_2) = \arg z_1 + \arg z_2$
   b) $\arg(z_1/z_2) = \arg z_1 - \arg z_2$
   c) $\arg z = \text{Arg } z - 2\pi n$
   d) $\arg(z^n) = n \arg z$

16. The principal argument of $-\sqrt{3}-i$
   a) $\pi/2$     b) $\pi/6$     c) $-5\pi/6$     d) $\pi/6$

17. The principal argument of $(1+i\sqrt{3})^2/4i(1-i\sqrt{3})$ is
   a) $8\pi/3$     b) $\pi/6$     c) $5\pi/6$     d) $\pi/2$

18. The solution of equation $|z| - z = 1+2i$ is
   a) $3/2 - 2i$
   b) $-3/2 + 2i$
   c) $2-3/2i$
   d) $2+3/2i$

19. The product of all four values of $(\cos \pi/3 + i\sin \pi/3)^{3/4}$ is
   a) $-1$     b) $i$     c) $-i$     d) $1$

20. If $\omega \neq 1$ is a cube root of unity and
   $$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 & 1 \\ 1 & \omega^2 & \omega^7 \end{bmatrix} = 3k$$
   then $k$ is equal to
   a) $-1$     b) $1$     c) $-\sqrt{3}i$     d) $\sqrt{3}i$

**Note**: i) Answer any seven questions.
   ii) Question No. 30 is compulsory

**SECTION – B**

7 x 2 = 14
SECTION – C

Note: i) Answer any seven questions.

ii) Question No. 40 is compulsory
31. Simplify i) \( \sum_{n=1}^{102} (i^n) \) ii) \( \sum_{n=1}^{10} (i^{n+50}) \)
32. For what values of \( x \) and \( y \), the numbers \( -3 + ix^2 \) and \( x^2 + y + 4i \) are complex conjugate of each other
33. Show that \((2+i\sqrt{3})^2 + (2-i\sqrt{3})^2\) is purely imaginary
34. The complex numbers \( u, v, w \) are related by \( \frac{1}{u} = \frac{1}{v} + \frac{1}{w} \). If \( v = 3-4i \) and \( w = 4+3i \), find \( u \) in rectangular form
35. If \( z_1, z_2, z_3 \) be complex numbers such that \( |z_1| = r, |z_2| = r, |z_3| = r \) prove that \( \left| \frac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_1 + z_2 + z_3} \right| = r \)
36. For any complex number \( z_1, z_2 \) such that \( |z_1| = |z_2| = 1 \) and \( z_1 z_2 \neq 1 \) then show that \( z_1 + z_2 / 1 + z_1 z_2 \) is a real number
37. If \( z = x + iy \) such that \( \text{Im}\left[ \frac{2z+1}{iz+1} \right] = 0 \), then find the locus of \( z \)
38. Obtain cartesian form i) \( |z - 4| = 3 \) ii) \( \text{Im}\left[ (1-i)z + 1 \right] = 0 \)
39. Find quotient in rectangular form
\[
\frac{2(\cos\frac{9\pi}{4} - i\sin\frac{9\pi}{4})}{4(\cos\frac{-3\pi}{2} + i\sin\frac{-3\pi}{2})}
\]
40. \( \frac{1+z}{1-z} = \cos 2\theta + i\sin 2\theta \), show that \( z = itan\theta \)

SECTION – D

Note: i) Answer All the questions.

41) a) Show that the points 7+9i, -3+7i, 3+3i form a right angled triangle on the argand diagram and show that \( z = 3 + 2i \) represent complex numbers \( z, iz \) and \( z+iz \) forms an equilateral triangle
   Or
   b) Show that the points 2i, 1+i, 4+4i, 3+5i form a rectangle on the argand diagram

42) a) If \( |z - \frac{2}{z}| = 2 \) show that greatest value and least of \( |z| \) are \( \sqrt{3} + 1 \), \( \sqrt{3} - 1 \) and Show that the equation \( z^3 + 2\overline{z} = 0 \) has five solution
   Or
   b) Find polar form for \( \frac{i-1}{(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})} \) and \( -2 - 2i \)

43) a) Find locus if \( \arg \left( \frac{z-1}{z+3} \right) = \pi/2 \)
b) Simplify \((-1 + i)^{18}\)

44) a) Show that \(\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5 = -\sqrt{3}\)

Or

b) Find all cube roots of \(\sqrt{3} + i\)

45) a) \(2\cos\alpha = x + 1/x\) and \(2\cos\beta = y + 1/y\) then show that \(x^m y^n + 1/x^m y^n = 2\cos(m\alpha + n\beta)\) and \(z = \cos\theta + isin\theta\) show that \(z^n + 1/z^n = 2\cos n\theta\) and \(z^n - 1/z^n = 2isin n\theta\)

Or

b) Find fourth unit of unity
1. The fourth roots of unity are
   a) $\omega _1^\pm 1$  
   b) $\omega _2^\pm i$  
   c) $\omega _3^\pm -1$  
   d) $\omega _4^\pm -i$
2. If $p + qi = (2-3i)(4+2i)$
   a) $2Re(Z)$  
   b) $Re(Z)$  
   c) $Im(Z)$  
   d) $2Im(Z)$
3. The values of $\arg(Z)$ are
   a) $0$  
   b) $\pi$  
   c) $\pi/2$  
   d) indeterminate
4. If $Z = 0$ then the arg($Z$) is
   a) $0$  
   b) $90^0$  
   c) $180^0$  
   d) indeterminate
5. The modulus of $\sqrt{5} + \sqrt{3}i$ is
   a) $4$  
   b) $-4$  
   c) $-2\sqrt{2}$  
   d) $2\sqrt{2}$
6. Which of the following are correct?
   i) $Re(Z) = |Z|$  
   ii) $Im(Z) = |Z|$  
   iii) $|\bar{Z}| = |Z|$  
   iv) $|Z^n| = (|Z|)^n$
   a) (i), (ii)  
   b) (i), (iii)  
   c) (ii), (iii)  
   d) (i), (iii) and (iv)
7. The values of $\bar{Z} + \bar{Z}$ are
   a) $|Z|$  
   b) $2|Z|$  
   c) $-2|Z|$  
   d) $|Z|^2$
8. If $\omega$ is a cube root of unity then
   a) $\omega^2 = 1$  
   b) $\omega = 1$  
   c) $1 + \omega^2 = 0$  
   d) $1 - \omega^2 = 0$
9. The principal value of $\arg(Z)$ lies in the interval
   a) $[0, \pi/2]$  
   b) $(-\pi, 0]$  
   c) $[\pi/2, \pi]$  
   d) $[-\pi/2, \pi/2]$
10. The fourth roots of unity are
    a) $1, 1 \pm i, 1 \pm i$  
    b) $\pm 1, \pm i$  
    c) $1, 1 \pm i$  
    d) $1, -1$
11. The cube roots of unity are
    a) $\sqrt[3]{2}, 1 \pm \sqrt[3]{i}$  
    b) $1, 1 \pm \sqrt[3]{i}$  
    c) $1, 1 \pm \sqrt[3]{i}$  
    d) $-1$
12. The value of $e^{i\theta} - e^{-i\theta}$ is
    a) $\sin \theta$  
    b) $\cos \theta$  
    c) $2\sin \theta$  
    d) $2\cos \theta$
13. Identify the correct statement
    a) Sum of the moduli of two complex numbers is equal to their modulus of the sum
    b) Modulus of the product of the complex numbers is equal to sum of the moduli
    c) Arguments of the product of two complex numbers is the product of their arguments
    d) Arguments of the product of two complex numbers is equal to the sum of their arguments
14. If $a = 3 + i$ and $z = 2 - 3i$ then the points on the Argand diagram representing $az, 2az$ and $-az$ are
    a) Vertices of a right angled triangle
    b) Vertices of an equilateral triangle
    c) Vertices of an isosceles triangle
    d) Collinear
15. The modulus and amplitude of the complex number $3 + i$ are
    a) $\sqrt{3}$, $\pi/4$  
    b) $\sqrt{3}$, $\pi/2$  
    c) $\sqrt{3}$, $-\pi/4$  
    d) $\sqrt{3}$, $-\pi/2$
16. The value of $\left(\frac{1 + \sqrt{3}i}{2}\right)^{100}$ is
    a) $2$  
    b) $0$  
    c) $-1$  
    d) $1$
23. If \( z_n = \cos \frac{n\pi}{3} + i \sin \frac{n\pi}{3} \), then \( z_1 z_2 \ldots z_n \)
   a)1 b)-1 c)i d)-i
24. The conjugate of \( i^3 + i^4 + i^5 + i^6 \) is
   a)1 b)-1 c)0 d)-i
25. Identify the wrong identity
   a)\( z^n z^m = z^{m+n} \) b) \( z^{m/n} z^n = z^{m-n} \)
   c) \( (z^n)^m = z^{mn} \) d) \( (z_1 z_2)^m = z_1^m + z_2^m \)

26. If \( \frac{z + 3}{z - 5i} = \frac{1 + 4i}{2} \) then \( z \) is
   a)2-3i b)-2.3i c)-2 d)2+3i
27. The points \( i, -2 + i, \) and 3 is farthest from the origin?
   a) \( i \) b) 3 c)-2+i d)none of these
28. The square root of 6-8i is
   a) \( \sqrt{2} (-2 - i) \) b) \(-\sqrt{2} (-2 - i) \)
   c) \( \sqrt{2} (2 - i) \) d) \( \sqrt{2} (-2 + i) \)
29. If \( z = 3 + 2i \), then \( z, iz, z+i \)
   a)scalene triangle b)equilateral triangle
c)isosceles right angle d)obtuse triangle
30. The locus of \( |2z - 3 - i| = 3 \) is
   a)ellipse b)parabola
c)hyperbola d)circle
31. Match the following
   Z \ Argz
   1 \ \pi
   -1 \ -\pi/2
   i \ 0
   -i \ \pi/2
32. If \( \arg(z-1/z+1) = \pi/2 \) then locus of \( z \) is
   a)ellipse b)parabola
c)hyperbola d)circle
33. Identify the wrong identity
   a) \( (\cos \theta - i \sin \theta) = \cos n\theta - i \sin n\theta \)
   b) \( (\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta \)
c) \( (\cos \theta - i \sin \theta)^n = \cos n\theta + i \sin n\theta \)
   d) \( \sin \theta - i \cos \theta = i(\cos \theta - i \sin \theta) \)
34. The value of \( (\sin \pi/6 + i \cos \pi/6)^{18} \) is
   a)0 b)-1 c)i d)1
35. The value of \( (1+i)^{18} \) is
   a)512 b)-512 c)512i d)-512i
36. \( |z| = 1 \) represents a circle, the radius of circle is
   a)-1 b)2 c)4 d)1
37. Identify the wrong statement
   a) All the \( n \) roots of \( n \)th roots unity are in Geometrical Progression
   b) Sum of the \( n \) roots of \( n \)th roots unity always equals to zero.
   c) Product of the \( n \) roots of \( n \)th roots unity is equal to \((-1)^{n-1}\)
   d) \( n \) roots have arguments that differ by \( \pi/n \)
38. 1- \( \omega^n / 1 + \omega^n = \)
   a)1 b)-1 c)0 d)i
39. The cube roots of unity form a vertices
   a)scalene triangle b)equilateral triangle
c)isosceles right angle d)obtuse triangle
40. If \( |z - \frac{3}{n}| = 2 \) then greatest value of \( |z| \)
   a)-1 b)1 c)3 d)2
41. If \( |z_1| = 1, |z_2| = 2, |z_3| = 3 \), modulus of
   \( 9 z_1 z_2 + 4 z_1 z_3 + z_1 z_3 \) is 6 then \( |z_1 + z_2 + z_3| \) is
   a)2 b)3 c)4 d)1
42. If \( \alpha \) and \( \beta \) are the roots of \( x^2 + x + 1 = 0 \)
   then \( \alpha^{2021} + \beta^{2021} \) is
   a)1 b)-1 c)2 d)-1
43. The product of all four values of
   \( (\cos \pi/3 + i \sin \pi/3)^{3/4} \) is
   a)-1 b)i c)-i d)1
44. If \( z_1 = a + ib \) and \( z_2 = -a + ib \) lies on
   a)line \( y = -x \) b) line \( y = x \)
c) imaginary axis d)real axis
45. The fourth roots of unity form a vertices of
   a)Square b)hexagon
c)rectangle d)circle