I. Applications of Matrices and Determinants:
1. Rank of a matrix \( \rho(A) \):
   (i) The rank of a matrix \( A \) is the order of the largest non zero minor of \( A \).
   (ii) If \( A \) is a matrix of order \( m \times n \) then
       \[ \rho(A) < \min\{m, n\} \]
2. Determinant method (Cramer’s rule):
   When \( \Delta \neq 0 \), the unique solution is given by
   \[ x = \frac{\Delta x}{\Delta}, \quad y = \frac{\Delta y}{\Delta}, \quad z = \frac{\Delta z}{\Delta} \]
3. Transition probability matrix:
   (i) At equilibrium, \( (A)^T (B) = (A B) \)

II. Integral Calculus – I
1. Properties of indefinite integrals:
   (i) \[ \int a f(x) \, dx = a \int f(x) \, dx \]
   (ii) \[ \int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx \]
2. Standard results of indefinite integrals:
   1. \[ \int x^n \, dx = \frac{x^{n+1}}{n+1} + c \]
   2. \[ \int e^x \, dx = e^x + c \]
   3. \[ \int \sin x \, dx = -\cos x + c \]
   4. \[ \int \sec^2 x \, dx = \tan x + c \]
   5. \[ \int \frac{1}{x} \, dx = \log |x| + c \]
   6. \[ \int a^x \, dx = \frac{1}{\log a} a^x + c, a > 0 \text{ and } a \neq 1 \]
   7. \[ \int \cos x \, dx = \sin x + c \]
   8. \[ \int \csc^2 x \, dx = -\cot x + c \]
   9. \[ \int [f(x)]^n f'(x) \, dx = \frac{[f(x)]^{n+1}}{n+1} + c, n \neq -1 \]
   10. \[ \int \frac{f'(x)}{f(x)} \, dx = \log |f(x)| + c \]
   11. \[ \int uv \, dx = \int f(x) \, dx + \int g(x) \, dx \]
   12. \[ \int \frac{dx}{a^2 - x^2} = \int \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c \]
   13. \[ \int \frac{dx}{\sqrt{x^2 - a^2}} = \log |x + \sqrt{x^2 - a^2}| + c \]
   14. \[ e^x \left[ f(x) + f'(x) \right] \, dx = e^x f(x) + c \]

15. \[ \int \frac{f'(x)}{\sqrt{f(x)}} \, dx = 2 \sqrt{f(x)} + c \]
16. \[ \int u dv = uv - \int v du \]
17. \[ \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \]
18. \[ \int \frac{dx}{x^2 + a^2} = \log |x + \sqrt{x^2 + a^2}| + c \]
19. \[ \int e^x \left[ a f(x) + f'(x) \right] \, dx = e^x f(x) + c \]
20. \[ \int \sqrt{x^2 - a^2} \, dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + c \]
21. \[ \int \sqrt{x^2 + a^2} \, dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + c \]
3. Definite integral:
   \[ \int_a^b f(x) \, dx = F(b) - F(a) \]
4. Properties of definite integrals:
   1. \[ \int_a^b f(x) \, dx = \int_a^b f(t) \, dt \]
   2. \[ \int_a^b [f(x) \pm g(x)] \, dx = \int_a^b f(x) \, dx \pm \int_a^b g(x) \, dx \]
   3. \[ \int_a^b f(x) \, dx = -\int_a^c f(x) \, dx \]
   4. \[ \int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx \]
   5. \[ \int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx \]
   6. \[ \int_a^b f(x) \, dx = \int_a^b f(a-x) \, dx \]
   7. a) If \( f(x) \) is an even function, then \[ \int_{-a}^a f(x) \, dx = 2 \int_0^a f(x) \, dx \]
   b) If \( f(x) \) is an odd function, then \[ \int_{-a}^a f(x) \, dx = 0 \]
5. Particular case of Gamma Integral:
   If \( n \) is a positive integer, then \[ \int_0^\infty x^n e^{-ax} \, dx = \frac{n!}{a^{n+1}} \]
6. Properties of gamma function:
1. \( \Gamma(n) = (n-1)\Gamma(n-1), n > 1 \)
2. \( \Gamma(n+1) = n! \), \( n \) is a positive integer
3. \( \Gamma(n+1) = n\Gamma(n), n > 0 \)
4. \( \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \)
7. Definite integral as the limit of a sum:
\[
\int_{a}^{b} f(x) \, dx = \lim_{n \to \infty} \sum_{r=1}^{n} h \cdot f(a + rh) \quad \text{where} \quad h = \frac{b-a}{n}
\]
Note:
(i) \( 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} = \sum_{r=1}^{n} r \)
(ii) \( 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6} = \sum_{r=1}^{n} r^2 \)
(iii) \( 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[ \frac{n(n+1)}{2} \right]^2 = \sum_{r=1}^{n} r^3 \)

III. Integral Calculus – II
1. Area \( A = \int_{a}^{b} y \, dx \)
2. Area \( A = \int_{a}^{b} -y \, dx \)
3. Area \( A = \int_{c}^{d} x \, dy \)
4. Area \( A = \int_{c}^{d} -x \, dy \)
5. Area between two curves
\[
A = \int_{a}^{b} [f(x) - g(x)] \, dx.
\]
   i) Total sale \( = \int_{0}^{r} f(t) \, dt, 0 \leq t \leq r \)
2. Elasticity of demand is \( \eta_{d} = \frac{-p}{x} \frac{dx}{dp} \)
3. Total inventory carrying cost \( = c_{1} \int_{0}^{r} f(t) \, dt \)
4. Amount of annuity after \( N \) Payment is \( A = \int_{0}^{N} p e^{rt} \, dt \)
5. Cost function is \( C = \int (MC) \, dx + k \)
6. Average cost function is \( AC = \frac{C}{x} \)
7. Revenue function is \( R = \int (MR) \, dx + k \)
8. Demand function is \( P = \frac{R}{x} \)
9. Profit function is \( = MR - MC = r(x) - C(x) \)
10. Consumer’s surplus \( = \int_{0}^{x_{0}} f(x) \, dx - x_{0}p_{0} \)
11. Producer’s surplus \( = x_{0}p_{0} - \int_{x_{0}}^{x} p(x) \, dx \)

IV. Differential Equations
1. Variables are separable \( f(x) \, dx = g(y) \, dy \) \( \text{or} \) \( f(x) \, dx + g(y) \, dy = 0 \)
   By direct integration we get the solution.
2. Homogeneous Differential Equations
   \( \frac{dy}{dx} = F\left(\frac{y}{x}\right) \)
   Put \( y = vx \) and \( \frac{dy}{dx} = v + x \frac{dv}{dx} \)
   The given differential equation becomes \( v + x \frac{dv}{dx} = F(v) \)
   Separating the variables, we get
   \( x \frac{dv}{dx} = F(v) - v \Rightarrow \frac{dv}{F(v) - v} = \frac{dx}{x} \)
   By integrating we get the solution in terms of \( v \) and \( x \).
   Replacing \( v \) by \( \frac{y}{x} \) we get the solution.
3. Linear diff. equations of first order:
   i) \( \frac{dy}{dx} + Py = Q \) \then
   \( ye^{\int P \, dx} = \int Qe^{\int P \, dx} \, dx + c \)
   ii) \( \frac{dx}{dy} + Px = Q \) \then
   \( xe^{\int P \, dy} = \int Qe^{\int P \, dy} \, dy + c \)
4. Second Order first degree differential equations with constant coefficients:  
General solution is \( y = C.F + P.I \)

<table>
<thead>
<tr>
<th>Nature of roots</th>
<th>Complementary function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real and different ((m_1 \neq m_2))</td>
<td>( Ae^{m_1x} + Be^{m_2x} )</td>
</tr>
<tr>
<td>Real and equal (m_1 = m_2 = m)(say)</td>
<td>((Ax - B)e^{mx})</td>
</tr>
<tr>
<td>Complex roots ( (\alpha \pm i\beta) )</td>
<td>( e^{\alpha x}(A\cos \beta x + B\sin \beta x) )</td>
</tr>
</tbody>
</table>

V. Numerical Methods
1. \( \Delta f(x) = f(x + h) - f(x) \)
2. \( \Delta f(x) = f(x) - f(x - h) \)
3. \( \Delta f(x + n) = \Delta f(x) \)
4. \( \Delta f(x) = f(x + h) \)
5. \( P^n f(x) = f(x + nh) \)
6. Newton's forward interpolation formula:  
\[
y(x=x_0+nh) = y_0 + \frac{n}{1!} \Delta y_0 + \frac{n(n-1)}{2!} \Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!} \Delta^3 y_0 + \ldots
\]
7. Newton's backward interpolation formula:  
\[
y(x=x_0+nh) = y_n + \frac{n}{1!} \Delta y_n + \frac{n(n+1)}{2!} \Delta^2 y_n + \frac{n(n+1)(n+2)}{3!} \Delta^3 y_n + \ldots
\]
8. Lagrange's interpolation formula:  
\[
y = f(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)} y_0
+ \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)} y_1 + \ldots
+ \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})} y_n
\]

VI. Random Variable and Mathematical Expectation
1. Probability Mass function:  
\[
P_x(x) = p(x) = \begin{cases} p(X=x_i) = p_i = p(x_i) & \text{if } x-x_i, i=1,2,\ldots,n, \ldots \\ 0 & \text{if } x \neq x_i \end{cases}
\]
It Must satisfy the following condition  
(i) \( p(x_i) \geq 0 \) \( \forall i \),  
(ii) \( \sum_{i=1}^{\infty} p(x_i) = 1 \)
2. Discrete distribution function:  
\[
F_X(x) = P(X \leq x), \text{ for all } x \in R
\]
i.e., \( F_X(x) = \sum_{x_i \leq x} p(x_i) \)
3. Probability density function:  
\[
P(t_1 \leq X \leq t_2) = \int_{t_1}^{t_2} f_X(x)dx.
\]
It Must satisfy the following condition  
(i) \( f(x) \geq 0 \ \forall \ x \) and  
(ii) \( \int_{-\infty}^{\infty} f(x)dx = 1 \).

4. Continuous distribution function  
(The distribution function (d.f) or The cumulative distribution function (c.d.f))

The function \( F_X(x) \) or simply \( F(x) \) has the following properties  
(i) \( 0 \leq F(x) \leq 1, \ -\infty < x < \infty \)
(ii) \( F(-\infty) = \lim_{x \to -\infty} F(x) = 0 \) and \( F(\infty) = \lim_{x \to \infty} F(x) = 1. \)
(iii) \( F() \) is a monotone, non-decreasing function; that is, \( F(a) \leq F(b) \) for \( a < b \).
(iv) \( F() \) is continuous from the right; that is, \( \lim_{h \to 0} F(x+h) = F(x) \).
(v) \( F'(x) = \frac{d}{dx} F(x) = f(x) \geq 0 \)
(vi) \( F'(x) = \frac{d}{dx} F(x) = f(x) \Rightarrow dF(x) = f(x)dx \)
\( dF(x) \) is known as probability differential of \( X \).
(vii) \( P(a \leq x \leq b) = \int_{a}^{b} f(x)dx = \int_{a}^{b} f(x)dx - \int_{-\infty}^{a} f(x)dx = \int_{-\infty}^{b} f(x)dx - \int_{-\infty}^{a} f(x)dx = F(b) - F(a) \)
5. Mathematical Expectation:
   i) \( X \) be a discrete random variable then  
   \( E(X) = \sum x p(x) \)
   ii) \( X \) is a continuous random variable then  
   \( E(X) = \int x f(x)dx \)
   iii) The mean of \( X \), denoted by \( \mu_x \) or \( E(X) \).
6. Variance:
   i) The variance of \( X \) is defined by  
   \( Var(X) = \sum (x - E(X))^2 p(x) \)
   if \( X \) is discrete random variable with probability mass function \( p(x) \).
   ii) \( Var(X) = \int (x - E(X))^2 f_X(x)dx \)
   if \( X \) is continuous random variable with probability density function \( f_X(x) \).
   iii) Expected value of \( [X - E(X)]^2 \) is called the variance of the random variable.  
   i.e.,  
   \( Var(X) = E[X - E(X)]^2 = E[X^2] - [E(X)]^2 \)
The normal probability distribution is given by

\[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2} \]

where \( \mu = \text{mean} \) and \( \sigma = \text{standard deviation} \).

2. Poisson distribution:
   
   i) The Poisson probability distribution is
   
   \[ p(x) = \frac{e^{-\lambda} \lambda^x}{x!} \]
   
   Where \( x = 0, 1, 2, \ldots \) and \( \lambda = np \).
   
   ii) The mean and variance of the Poisson distribution is \( \lambda \).
   
   iii) Poisson distribution can never be symmetrical.

3. Normal distribution:
   
   i) The normal probability distribution is given by
   
   \[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2} \]
   
   ii) In normal distribution the mean, median and mode are equal.
   
   iii) Standard normal random variate is denoted as \( Z = (X - \mu) / \sigma \).
   
   iv) The standard normal probability distribution is
   
   \[ \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \]

VIII. Sampling Techniques and Statistical Inference

1. Test of significance for single mean:
   
   i) The test statistic (for large samples) is:
   
   \[ Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \]
   
   2. Confidence limits:
   
   \[ \bar{X} - \frac{Z_{\alpha/2} \sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + \frac{Z_{\alpha/2} \sigma}{\sqrt{n}} \quad \text{or} \quad x - \frac{Z_{\alpha/2} \sigma}{\sqrt{n}} < \mu < x + \frac{Z_{\alpha/2} \sigma}{\sqrt{n}} \]